Final exam sheet for 'Atmospheric Dynamics'.

Questions

1. (6 Points)

   The quasi-geostrophic vorticity equation is given by

   $$\frac{d_g \xi_g}{dt} = -f_0 \left(\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} \right) - \beta v_g \quad , \tag{1}$$

   with

   $$\xi_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} = \frac{1}{f_0} \nabla^2 \Phi \tag{2}$$

   and

   $$\mathbf{v}_g = f_0^{-1} \mathbf{k} \times \nabla \Phi \quad . \tag{3}$$

   Assume that the geopotential field is given by

   $$\Phi(x, y, p, t) = \Phi_0(p) + f_0[-Uy + k^{-1} V \cos(\pi p / p_0) \sin k(x - ct)] \quad , \tag{4}$$

   where U, V, c, k, p_0 are constants.

   a) Use the quasi-geostrophic vorticity equation (1) to obtain the horizontal divergence field, assuming that $df/dy = \beta$ is a constant (not zero).

   b) Assuming that the vertical velocity in pressure coordinates, ω, fulfills $\omega(p_0) = 0$, obtain an expression for $\omega(x, y, p, t)$ by integrating the continuity equation in pressure coordinates with respect to pressure.

2. (10 points)

   The thermal wind equation in the pressure coordinate system is

   $$\frac{\partial \mathbf{v}}{\partial lnp} = -\frac{R}{f} \mathbf{k} \times \nabla T \tag{5}$$

   a) Which two basic approximate balances valid for large-scale motion lead to the thermal wind equation 5? Write the equations for these balances and derive equation 5.

b) Bob lives at a latitude of 45 degrees north. He wakes up one morning and measures a temperature of 10 C and no wind. His friend Bill lives 1000 km east and measures a temperature of 0 C and no wind as well. When Bob and Bill look up to the sky they see clouds. Assume that the horizontal temperature changes are constant with height. Are the clouds moving south or north? Assuming that the clouds are at 500 hPa and the surface pressure is 1000 hPa how fast are they moving? Show all your work and state your assumptions carefully. (The gas constant for air is $R = 287$ J K^{-1} kg^{-1}).

c) Calculate the temperature advection by the thermal wind at 500 hPa $\mathbf{v_T} \cdot \nabla T$ that you calculated in b), assuming ∇T to be constant with height.

3. (10 Points)

Consider the conservation of barotropic potential vorticity

$$\frac{d_h}{dt}\frac{(\xi + f)}{h} = 0 \quad . \tag{6}$$

a) State the conditions under which Eq. (6) becomes the barotropic vorticity equation

$$\frac{d_h(\xi + f)}{dt} = 0 \quad . \tag{7}$$

b) Using the approximations $\xi = \nabla^2\Phi/f_0$ and $\mathbf{v} = f_0^{-1}\mathbf{k} \times \nabla\Phi$ show that the barotropic vorticity equation (7) can be written in the form

$$\frac{d_h}{dt}\nabla^2\psi + \beta\frac{\partial\psi}{\partial x} = 0 \quad . \tag{8}$$

c) Linearize Eq. (8) around a basic state that fulfills $\overline{\psi} = -\overline{u}y + const.$, where $\overline{u}$ is constant and assuming only zonally and time dependent streamfunction perturbations $\psi'(x,t)$. Show that the linearized barotropic vorticity equation is given by

$$\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right)\frac{\partial^2\psi'}{\partial x^2} + \beta\frac{\partial\psi'}{\partial x} = 0 \quad , \tag{9}$$

where β is assumed to be constant.

d) Derive the phase speed of zonally propagating Rossby waves (assume solutions of type: $\psi' = Ae^{i(kx - \nu t)}$). What is the phase speedy if $\overline{u} = 10$ m/s, $\beta = 1 \cdot 10^{-11}\mathrm{s}^{-1}\mathrm{m}^{-1}$, and the wavelength is $L_x = 3.14 \cdot 10^6$ m? (assume $\pi \approx 3.14$)

e) Calculate the wavelength of stationary Rossby waves. Which wavelength results for $\beta = 1 \cdot 10^{-11}\mathrm{s}^{-1}\mathrm{m}^{-1}$, and $\overline{u} = 10$ m/s (assume $\pi \approx 3.14$)?

f) Calculate the group velocity of purely zonally propagating Rossby waves and the group velocity for the special case of stationary Rossby waves.

g) What modifications in the vorticity equation 8 and in the basic state introduced in c) are needed to support baroclinic instability? What other equation is needed to close the system in that case?

h) A stationary Rossy wave generated by an equatorial heating is moving northeastward. Assuming the zonal stationary wavenumber to be constant, explain why there is a latitude where the stationary meridional wavenumber becomes 0. What is the name of this latitude?

4. (6 points)

a) For a motionless stationary state along the equator, the following equation for the thermocline is valid:

$$\frac{\partial h}{\partial x} = \frac{1}{\rho h g'} \tau_x \quad . \tag{10}$$

Explain the meaning of all variables and constants in this equation.

b) Assume a mean wind stress distribution along the equator:

$$\begin{aligned} \tau_x &= 0 \quad \text{for lon} \leq 180\text{ E} \\ \tau_x &= -0.05\ \mathrm{N/m^2} \quad \text{for } 180\text{ E} \leq \text{lon} \leq 240\text{ E} \\ \tau_x &= 0 \quad \text{for lon} \geq 240\text{ E} \end{aligned}$$

Using the approximation 10, calculate the thermocline distribution along the equator. Assuming that the thermocline depth at the western edge is 100 m, what is the approximate total change in height between 180 E and 240 E? (Assume the radius of the Earth is $r = 6.37 \cdot 10^6$ m).

c) In the eastern equatorial Pacific a warm Sea Surface temperature anomaly is observed that develops into a El Nino event. Explain the atmospheric adjustment processed that follow and eventually lead to the positive atmospheric feedback mechanism. Explain also the positive ocean feedback mechanism that follows. What is the name of this positive atmosphere-ocean coupled feedback mechanism?

5. (4 points)

The stationary equation for the vertical component of the zonal mean flow is

$$-S_p\overline{\omega} = -\frac{\partial \overline{v'T'}}{\partial y} + \frac{\overline{Q}}{c_p} \tag{11}$$

a) Explain the meaning of each term in this equation.

b) What are the main terms responsible for the Hadley Circulation. Draw the Hadley circulation in a latitude-vertical section. Indicate where rising and sinking motion can be found.

6. (4 points)

Let x_{ij} be a model variable at a certain gridpoint at times $i = 1, ..., N$ for ensemble members $j = 1, ..., M$. Calculate from this and explain the meaning of the following 3 quantities:

a) the signal variance

b) the noise variance

c) the signal-to-noise ratio.

d) What is the range of values of the signal-to-noise ratio and what value may be considered as a threshold for a large predictability, and explain why.