Final exam sheet for 'Atmospheric Dynamics'.

Questions

1. (10 points)

   The thermal wind equation in the pressure coordinate system is

   $$\frac{\partial \mathbf{v}}{\partial lnp} = -\frac{R}{f}\mathbf{k} \times \nabla T \tag{1}$$

   a) Which two basic approximate balances valid for large-scale motion lead to the thermal wind equation 1? Write the equations for these balances and derive equation 1.

   b) Bob lives at a latitude of 45 degrees north. He wakes up one morning and measures a temperature of 10 C and no wind. His friend Bill lives 1000 km east and measures a temperature of 0 C and no wind as well. When Bob and Bill look up to the sky they see clouds. Assume that the horizontal temperature changes are constant with height. Are the clouds moving south or north? Assuming that the clouds are at 500 hPa and the surface pressure is 1000 hPa how fast are they moving? Show all your work and state your assumptions carefully. (The gas constant for air is $R = 287$ J K^{-1} kg^{-1}).

   c) Calculate the temperature advection by the thermal wind at 500 hPa $\mathbf{v_T} \cdot \nabla T$ that you calculated in b), assuming ∇T to be constant with height.

2. (10 points)

   a) The approximate horizontal momentum equations for the mean flow in the atmospheric boundary layer may be written as:

   $$\frac{\overline{d\overline{u}}}{dt} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} + f\overline{v} - \frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y} - \frac{\partial \overline{u'w'}}{\partial z} \tag{2}$$

   $$\frac{\overline{d\overline{v}}}{dt} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial y} - f\overline{u} - \frac{\partial \overline{v'u'}}{\partial x} - \frac{\partial \overline{v'v'}}{\partial y} - \frac{\partial \overline{v'w'}}{\partial z}\ , \tag{3}$$

   Explain (briefly) the meaning of all terms in this equations.

b) In the last 3 terms of the two momentum equations 2 and 3 only the terms containing $\overline{u'w'}$ and $\overline{v'w'}$ turn out to be relevant. What parameterization is typically used for these terms in order to derive the equations for the Ekman Layer? Write down the parameterization formulae and insert them into the momentum equations (dropping the overbars to indicate averaged quantities) and show that the approximate equations for the Ekman Layer can be written as (assuming the mean total derivatives of mean velocities du/dt and dv/dt to be negligible):

$$K_m \frac{\partial^2 u}{\partial z^2} + f(v - v_g) = 0 \quad (4)$$

$$K_m \frac{\partial^2 v}{\partial z^2} - f(u - u_g) = 0 \; , \quad (5)$$

c) Show by inserting in the equations 4 and 5 that

$$u = u_g \left[1 - \cos(\gamma z) e^{-\gamma z}\right] \quad (6)$$

$$v = u_g \sin(\gamma z) e^{-\gamma z} \; , \quad (7)$$

with $\gamma = \sqrt{f/(2K_m)}$ is a solution of the Ekman Equations, assuming that the geostrophic flow is zonal, $v_g = 0$. Sketch the solution 6, 7 in a diagram with v on the y-axis and u on the x-axis.

d) Calculate the horizontal divergence in the Ekman Layer from the solutions 6 and 7 (assuming the geostrophic wind to be non-divergent).

e) A vertical integration of the divergence leads to the approximate vertical velocity at the top of the boundary layer De

$$w(De) \approx \xi_g \sqrt{\frac{K_m}{2f}} \; . \quad (8)$$

Letting $\xi_g = 10^{-5}\mathrm{s}^{-1}$, $K_m = 4 \ \mathrm{m}^2 \ \mathrm{s}^{-1}$, $f = 10^{-4}\mathrm{s}^{-1}$, calculate the vertical velocity $w(De)$ at the top of the boundary layer.

3. (6 points)

a) For a motionless stationary state along the equator, the following equation for the thermocline is valid:

$$\frac{\partial h}{\partial x} = \frac{1}{\rho h g'} \tau_x \quad . \quad (9)$$

Explain the meaning of all variables and constants in this equation.

b) Assume a mean wind stress distribution along the equator:

$$\begin{aligned}
\tau_x &= 0 \quad \text{for lon} \leq 180 \text{ E} \\
\tau_x &= -0.05 \text{ N/m}^2 \quad \text{for } 180 \text{ E} \leq \text{lon} \leq 240 \text{ E} \\
\tau_x &= 0 \quad \text{for lon} \geq 240 \text{ E}
\end{aligned}$$

Using the approximation 9, calculate the thermocline distribution along the equator. Assuming that the thermocline depth at the western edge is 100 m, what is the approximate total change in height between 180 E and 240 E? (Assume the radius of the Earth is $r = 6.37 \cdot 10^6$ m).

c) In the eastern equatorial Pacific a warm Sea Surface temperature anomaly is observed that develops into a El Nino event. Explain the atmospheric adjustment processed that follow and eventually lead to the positive atmospheric feedback mechanism. Explain also the positive ocean feedback mechanism that follows. What is the name of this positive atmosphere-ocean coupled feedback mechanism?

4. (4 points)

The stationary equation for the vertical component of the zonal mean flow is

$$-S_p \overline{\omega} = -\frac{\partial \overline{v'T'}}{\partial y} + \frac{\overline{Q}}{c_p} \tag{10}$$

a) Explain the meaning of each term in this equation.

b) What are the main terms responsible for the Hadley Circulation. Draw the Hadley circulation in a latitude-vertical section. Indicate where rising and sinking motion can be found.

5. (4 points)

The balance equation for the mass-integrated available potential energy is

$$\int_\tau \frac{de_{ape}}{dt} d\tau = \int_\tau \left\{ \frac{RT}{p}\omega + \frac{T - T_0(s)}{T} q \right\} d\tau \quad , \tag{11}$$

a) Explain the terms in equation 11.

b) Explain why extratropical cyclones convert available potential energy into kinetic energy.

6. (3 points)

   A discrete sea surface temperature field has 100 gridpoint in zonal, 50 in meridional direction. It is measured at 60 time intervals. If you are asked to perform a principal component analysis, what is the dimension of the resulting principal components (PC) and the empirical orthotogonal functions (EOFs)? How many EOFs and PCs can be calculated?

7. (6 points)

   a) Any local precipitation event in a given region is governed by the moisture budget equation

   $$P = E - \int_0^{ps} \nabla_h \cdot (m_v \mathbf{v}_h) dp/g \quad . \tag{12}$$

   Explain all terms in this equation. What can be the sources for the precipitation in the event?

   b) Explain the Charney feedback mechanism between vegetation and rainfall, and how this mechanism contributes to a strengthening of the Sahel drought phenomenon. Draw a schematic.